tional loading ($\sigma_2 < \sigma_0$ during $0.5t_1$) occurs and then repeated unloading (the stress $\sigma_2 < \sigma_0$ is applied during τ until rupture). Integrating (6) for such loading, we obtain that the time τ of the stress σ_2 acting in the second cycle is $0.5t_2$. If a loading with many passages through σ_0 is considered, then it can be shown by an analogous method that rupture sets in because of the combined action of the stress σ_1 during the time t_1 and the stress σ_2 during t_2, where t_1 is the total time of application of the stress σ_1 in all the stages, and t_2 is the total time during which the stress σ_2 was applied. It hence follows that the sum of the partial times is independent of the quantity of passages of the stress through σ_0, and agrees with the value (9). Therefore, model (3) and (4) for the description of loading with single and multiple passages through the stress σ_0 results in a unilateral deviation ($1 < A < 2$) from the linear summation principle for the partial times.

Let us note that if the rupture condition (4) is replaced by

$$\min(\omega, \Omega) = 1, \tag{12}$$

then the deviation from condition (2) towards $A < 1$ can be described by using the model (3) and (12).

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PROBLEM OF NORMAL PRESSURE WAVES RUNNING AGAINST A STAMP

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The problem of the motion of a rigid massive circular stamp with a flat base under the action of oncoming normal pressure waves is examined. The stamp is assumed to be in frictionless contact with an elastic medium. It is assumed that the pressure wave is a plane wave and arrives from infinity. By removing the normal pressures from the surface of the medium by solving the boundary-value problem in the absence of the stamp (unmixed problem), the starting boundary-value problem reduces to the following mixed problem: a wave, which interacts with the stamp, travels along the surface of the medium screened from the normal pressure. Adding to the solution of this mixed problem the solution corresponding to the unmixed problem, we obtain the solution of the starting problem. Taking into account the fact that it is easy to solve the unmixed problem with the help of Fourier and Laplace integrals, in this work, we are primarily concerned with the mixed problem noted above with a screened surface outside the stamp.

1. We are studying the problem of the interaction of a rigid stamp with mass m, occupying a circular region Ω with radius a in a plane, with an elastic layered medium. It is assumed that the contact is frictionless, while a uniformly moving normal pressure pulse $p(x, y, t)$ acts on the stamp. It is necessary to find the normal component of the contact stresses $q(x, y, t)$, the vertical displacement of the center of the stamp $\delta(t)$, as well as the angles of its rotation relative to the horizontal axes $\omega(t)$ and $\theta(t)$; we shall determine $q(x, y, t)$ by solving the dynamic Lamb equation

$$(\lambda + 2\mu)\,\mathrm{grad\,div}\,\mathbf{U} - \mu\,\mathrm{rot\,rot}\,\mathbf{U} - \rho\partial^2\mathbf{U}/\partial t^2 = 0$$

with mixed boundary conditions and initial conditions. In particular, in the case of nonstationary action of the stamp on an elastic homogeneous half-space ($z \leqslant 0$), the boundary conditions have the form

$$\tau_{xz}(x, y, 0, t) = \tau_{yz}(x, y, 0, t) = 0, \ -\infty < x, y < +\infty,$$

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$$w(x, y, 0, t) = f(x, y, t),\ x, y \in \Omega,$$
$$\sigma_{zz}(x, y, 0, t) = 0,\ x, y \notin \Omega,$$

for $z \to -\infty$ $u(x, y, z, t), v(x, y, z, t), w(x, y, z, t) \to 0$, where f(x, y, t) is the displacement of a point on the rigid stamp with a flat base with coordinates (x, y, 0). In a Cartesian coordinate system, whose origin coincides with the center of the stamp, while the positive Oz axis coincides with the direction of the external normal to the surface of the medium, f(x, y, t) is described by the following equation:

$$f(x, y, t) = \delta(t) + \omega(t)y - \theta(t)x.$$

The initial conditions are assumed to be zero conditions, i.e.,

$$u(x, y, z, 0) = v(x, y, z, 0) = w(x, y, z, 0) = 0,$$
$$u_t'(x, y, z, 0) = v_t'(x, y, z, 0) = w_t'(x, y, z, 0) = 0.$$

By using the Fourier transformation with respect to coordinates and the Laplace transformation in time, the problem indicated reduces to solving an integral equation, depending on the Laplace transformation parameter s:

$$\frac{1}{4\mu}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_0(\alpha, \beta, s)\, e^{-i(\alpha x+\beta y)}d\alpha d\beta \int\int_{\Omega} \bar{q}(\xi, \eta, s)\, e^{\,i(\alpha\xi+\beta\eta)}d\xi d\eta = \bar{f}(x, y, s). \tag{1.1}$$

Here $\bar{q}(\xi, \eta, s)$ is the Laplace transformation of the contact pressures $q(\xi, \eta, t)$ with respect to time; $\bar{f}(x, y, s)$ is the Laplace transform of the vertical motion of the stamp f(x, y, t). We note that the function in the integrand $K_0(\alpha, \beta, s)$ for different types of layered media has the same form as in corresponding problems involving steady state oscillations [1] with the frequency of oscillations ω replaced by is (i is $\sqrt{-1}$). Transforming in (1.1) to polar coordinates and expanding the function

$$F(r, \varphi, s) = \bar{f}[x(r, \varphi), y(r, \varphi), s],\ Q(\rho, \gamma, s) = \bar{q}[\xi(\rho, \gamma), \eta(\rho, \gamma), s]$$

in a Fourier series with respect to φ and γ, we obtain equations for determining the Fourier coefficients:

$$\begin{gathered} F(r, \varphi, s) = c_0(s) + c_1(s)re^{i\varphi} + c_{-1}(s)re^{-i\varphi}, \\ \int_0^a k(r, \rho, s)\, Q_n(\rho, s)\, \rho d\rho = F_n(r, s),\ 0 \leqslant r \leqslant a, \\ k(r, \rho, s) = \int_0^{\infty} K(u, s)\, I_n(ur)\, I_n(u\rho)\, udu,\ n = -1, 0, 1, \end{gathered} \tag{1.2}$$

where $F_{-1}(r, s) = c_{-1}(s)r$; $F_0(r, s) = c_0(s)$; $F_1(r, s) = c_1(s)r$; $c_0(s) \equiv \bar{\delta}(s)$; $c_1(s)$ and $c_{-1}(s)$ are linearly related with the rotation angles ω(s) and $\bar{\theta}(s)$; $\bar{\delta}(s)$, $\bar{\omega}(s)$, $\bar{\theta}(s)$ are Laplace transforms of δ(t), ω(t), θ(t), respectively. The function K(u, s) has the properties established in [2]. Solving Eq. (1.2) for fixed values of the parameter s, using one of the well-developed methods for studying this equation [2], we find

$$Q(r, \varphi, s) = \sum_{k=-1}^{1} c_k(s)\, Q_k(r, \varphi, s).$$

Next, in order to solve the problem, we shall apply the Laplace transformation with respect to time to the equations of motion of the stamp with zero initial conditions and transform to polar coordinates:

$$\begin{gathered} ms^2\bar{\delta}(s) = \int\int_{\Omega} Q(r, \varphi, s)\, rdrd\varphi - \int\int_{\Omega} P(r, \varphi, s)\, rdrd\varphi, \\ I_1 s^2\bar{w}(s) = \int\int_{\Omega} Q(r, \varphi, s)\, r^2 \sin\varphi drd\varphi - \int\int_{\Omega} P(r, \varphi, s)\, r^2 \sin\varphi drd\varphi, \\ I_2 s^2\bar{\theta}(s) = -\int\int_{\Omega} Q(r, \varphi, s)\, r^2 \cos\varphi drd\varphi + \int\int_{\Omega} P(r, \varphi, s)\, r^2 \cos\varphi drd\varphi, \end{gathered} \tag{1.3}$$

where I_1 and I_2 are the moments of inertia of the stamp relative to the horizontal axes; $P(r, \varphi, s)$ is the Laplace-transformed change in the external action on the stamp $p(x, y, t)$ in polar coordinates. Substituting into (1.3) the expression found for $Q(r, \varphi, s)$, we obtain a system of algebraic equations, from which the constants $c_k(s)$ are completely determined:

$$ms^2c_0(s) = \sum_{k=-1}^{1} c_k(s) \iint_{\Omega} Q_k(r, \varphi, s)\, r dr d\varphi - \iint_{\Omega} P(r, \varphi, s)\, r dr d\varphi,$$

$$I_1 s^2 \sum_{m=1}^{2} \lambda_{1m} c_k(s) = \sum_{k=-1}^{1} c_k(s) \iint_{\Omega} Q_k(r, \varphi, s)\, r^2 \sin\varphi dr d\varphi - M_{1p}(s), \tag{1.4}$$

$$I_2 s^2 \sum_{m=1}^{2} \lambda_{2m} c_k(s) = \sum_{k=-1}^{1} c_k(s) \iint_{\Omega} Q_k(r, \varphi, s)\, r^2 \cos\varphi dr d\varphi - M_{2p}(s).$$

Here the sums on the left side are the angles of rotation of the stamp relative to the x and y axes in the Laplace transformation; $M_{av}(s)$ is the moment of the external force relative to the x axis (n = 1) or y axis (n = 2). By constructing the inverse Laplace transformation of $\bar{\delta}(s)$, $\bar{\omega}(s)$, and $\bar{\theta}(s)$, we obtain a solution of the problem formulated.

2. As an example, we shall examine the problem of the motion of a circular rigid stamp in a plane on an elastic half-space ($z \leqslant 0$) when an oncoming plane wave acts on it:

$$p(y, t) = \begin{cases} 0, & t \geqslant 2a/V, \\ Ae^{-\alpha(Vt-y-a)}[\alpha(Vt-y-a)]^n. \end{cases}$$

Equation (1.4) in this case reduces to the form

$$ms^2c_0(s) - 2\pi \int_0^a c_0(s) Q_0(r, s)\, r dr - 2\pi \int_0^a P_0(r, s)\, r dr,$$

$$ma^2s^2 i c_1(s) = -\frac{2\pi}{i} \int_0^a P_1(r, s)\, r^2 dr + \frac{2\pi}{i} \int_0^a c_1(s) Q_1(r, s)\, r^2 dr. \tag{2.1}$$

The functions $P_0(r, s)$ and $P_1(r, s)$ are determined based on the well-known properties of the Laplace and Fourier transformations and have the form

$$P_0(r, s) = k_1(s)J_0(irs/V) + k_2(s)J_0(i\alpha r) + k_3(s)J_1(i\alpha r)\alpha r/i,$$
$$P_1(r, s) = k_1(s)J_1(irs/V) - k_2(s)J_1(i\alpha r) + k_3(s)$$
$$[J_0(i\alpha r) - J_2(i\alpha r)]\alpha r/2i, \quad k_1(s) = A\alpha V e^{-as/V}(s + \alpha V)^{-2},$$
$$k_2(s) = -Ae^{-\alpha a}(s + \alpha V)^{-1}[\alpha V(s + \alpha V)^{-1} + \alpha a]e^{-2as/V},$$
$$k_3(s) = A(s + \alpha V)^{-1}e^{-\alpha a}e^{-2as/V},$$

where $J_0(z)$, $J_1(z)$ and $J_2(z)$ are Bessel functions and, in addition,

$$\int_0^a P_0(r, s)\, r dr = k_1(s)\, I_1(\alpha s/V)\, \alpha V/s + k_2(s)\, I_1(\alpha a)\, a/\alpha + k_3(s)\, a^2 I_2(\alpha a),$$

$$\int_0^a P_1(r,s)\, r^2 dr = i\{k_1(s)\, a^2V/sI_2(as/V) - k_2(s)\, a^2/\alpha I_2(\alpha a) - k_3(s)\, a^3[I_1(\alpha a) - 3/\alpha a I_2(\alpha a)]\},$$

$I_1(z)$ and $I_2(z)$ are the modified Bessel functions. Equation (1.2) was solved by the method of fictitious absorption [3], since it permits separating analytically the singularity of the contact pressure on the stamp boundary and, in addition, the integrals of $Q_0(r, s)$ and $Q_1(r, s)$ in (2.1) are found by quadrature. In order to construct the functions $\delta(t)$ and $\omega(t)$, which are the original functions for the Laplace transforms $\bar{\delta}(s)$ and $\bar{\omega}(s)$, respectively, we use A. N. Tikhonov's regularization method [4]. The behavior of the vertical displacement of the center of the stamp δ and its angle of rotation relative to the Ox axis ω in time was analyzed numerically on a computer as a function of the velocity V of the oncoming pressure wave with the following parameters: A = 1, α = 1, n = 1, ν = 0.3, $\rho = 2.5 \cdot 10^3$ kg/m^3, $\mu = 7.2 \cdot 10^4$ kg/m·sec^2, where ν is Poisson's coefficient, ρ is the density of the medium, and μ is the Lamb parameter of the given medium.

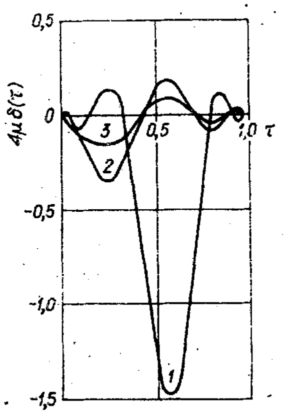

Fig. 1

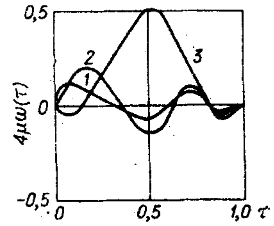

Fig. 2

The results of the calculations are presented in Figs. 1 and 2. The quantities $\delta(\tau)$ and $\omega(\tau)$ multiplied by 4μ are indicated along the vertical axes and, in addition, $\tau = t(1 + t)^{-1}$. This substitution permits studying the behavior of the functions indicated over the entire time interval. Curves 1-3 correspond to V = 1, 10, and 20 m/sec.

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STABILITY OF WELL WALLS

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1. Introduction. The scientific—technical problem of superdeep drilling is extremely difficult. Modern technology for constructing wells [1-3] consists of repeating the following cycle many times: drilling the bottom hole of a well with a special bit — extracting pieces of the fractured rock with a flushing liquid — wear or breakage of the drilling equipment and its replacement, usually including raising and lowering operations for the entire column of drillpipes.

Reinforcement of the walls of superdeep wells (exceeding 6 km) with casing columns becomes technically very complicated due to the loss of stability of the well walls, their collapse, and as a result, the large increase in the transverse cross section. In this case, the presence of the hydraulic pressure of the column of washing liquid serves as an important stabilizing factor. Clay and other additives in this liquid, by plugging pores, lead to the formation of a dense crust on the walls, hereby hermetically sealing the well. In what follows, we examine only vertical wells that are not protected by a casing column near the bottom hole at a distance, at least, of the order of 100 diameters of the well. Percolation of the liquid into the rock is neglected.

A very important factor under these conditions is the pressure from the above-lying rock. Considerable technological difficulties in superdeep drilling also arise from the increase in temperature (approximately by 20°C for each kilometer).

2. Local Instability of the Walls of a Circular Well. The well is a cylindrical cavity, $r < r_0$, $0 < z < H$ in the earth's crust $z < H$, where r and z cylindrical coordinates (z coin-

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